Quiz 10 (10pts)
Math 214 Section Q1 Winter 2010

Your name:________________________________ ID#:______________

Please, use the reverse side if needed.

1.(5 pts) Find the derivative of the function

$$f(x, y) = x^3 - 2xy + x + 3y$$

at $P_0(1, 1)$ in the direction of $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$. In what direction is the derivative of f at P_0 maximal?

Solution. Since $\mathbf{v}$ is not unit, we will find a unit vector in the direction of $\mathbf{v}$.

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}.$$

$$\nabla f = (3x^2 - 2y + 1)\mathbf{i} + (-2x + 3)\mathbf{j},$$

$$\nabla f\Big|_{(1,1)} = 2\mathbf{i} + \mathbf{j}.$$

Therefore,

$$(D_{\mathbf{u}} f)\Big|_{(1,1)} = \nabla f\Big|_{(1,1)} \cdot \mathbf{u} = \frac{11}{5}.$$

The derivative is maximal in the direction of ∇f, which is $\dfrac{2}{\sqrt{5}}\mathbf{i} + \dfrac{1}{\sqrt{5}}\mathbf{j}$.

2.(5 pts) Find an equation of the plane tangent to the surface

$$z = x^2 + 3xy + x\cos y + y$$

at the point $P_0(1, 0, 2)$.

Solution. The tangent plane is given by

$$z - 2 = z_x(1, 0)(x - 1) + z_y(1, 0)y$$

$$z_x = 2x + 3y + \cos y, \qquad z_y = 3x - x\sin y + 1$$

$$z_x(1, 0) = 3, \qquad z_y(1, 0) = 4.$$

So the plane has the equation

$$z - 2 = 3(x - 1) + 4y,$$

$$3x + 4y - z - 1 = 0.$$